$\int x^2 \sin^{-1}(x^{3/2})$ dx

Let t = $x^{3/2}$ then $t^{2/3}$ = x

dt = 3/2 $x^{1/2}$ dx

dx = 2/3 $x^{-1/2}$ dt = 2/3 $t^{-1/3}$ dt

$\int x^2 \sin^{-1}(x^{3/2})$ dx = $\int (t^{4/3}) \sin^{-1}(t)$ (2/3 $t^{-1/3}$ dt) = 2/3 $\int t \sin^{-1}(t) dt$ --- Eqn .. 1

Applying Integration by parts

Let I = $\int t \sin^{-1}(t) dt$ = $\sin^{-1}t \int t\ dt$ - $\int \frac{1}{\sqrt{1-t^2}} \frac{t^2}{2}$ dt

= $(\sin^{-1}t) \frac{t^2}{2}$ - 1/2 $\int \frac{1}{\sqrt{1-t^2}} \frac{t^2}{1}$ dt ------ Eqn 2

Let J = $\int \frac{t^2}{\sqrt{1-t^2}}$ dt

Let t = $\sin\theta$ then dt = $\cos\theta\ d\theta$

J = $\int \frac{(\sin\theta)^2 \cos\theta\ d\theta}{\cos\theta}$ = $\int (\sin\theta)^2\ d\theta$ = 1/2$\int (1-\cos 2\theta) d\theta$ = ½ (θ) - ¼ $\sin 2\theta$

J = ½ $(\sin^{-1}t)$ - ¼ $\sin(2(\sin^{-1}t))$ = ½ $(\sin^{-1}t)$ -1/2 [$\sin(\sin^{-1}t)\ \cos(\sin^{-1}t)$]

J = ½ $(\sin^{-1}t)$ –(½) $t\sqrt{1-t^2}$ + C

Substituting J in Eqn 2

I = $(\sin^{-1}t) \frac{t^2}{2}$ - 1/4$(\sin^{-1}t)$ + ¼ $t\sqrt{1-t^2}$ = $(\sin^{-1}t)$ $(\frac{2t^2-1}{4})$ + ¼ $t\sqrt{1-t^2}$ + C

Substituting t value in I

I = $(\sin^{-1}x^{3/2})$ $(\frac{2x^3-1}{4})$ + ¼ $x^{3/2}\sqrt{1-x^{3/2}}$

Substituting I value in Eqn ..1

$\int x^2 \sin^{-1}(x^{3/2})$ dx =2/3 [$(\sin^{-1}x^{3/2})$ $(\frac{2x^3-1}{4})$ + ¼ $x^{3/2}\sqrt{1-x^{3/2}}$] + C

= $(\sin^{-1}x^{3/2})$ $(\frac{2x^3-1}{6})$ + 1/6 $x^{3/2}\sqrt{1-x^{3/2}}$ + C